Sign Test Example p. 448

$H_0 : \tilde{\mu} = 98$

$H_a : \tilde{\mu} > 98$

$\alpha = 0.01$

We replace each value with a + sign if it is greater than 98, - sign if less than 98, or we remove the data if it equuals 98.

	Sign
99	+
102.3	+
99.8	+
100.5	+
99.7	+
96.2	−
99.1	+
102.5	+
103.3	+
97.4	−
100.4	+
98.9	+
98.3	+
98.0	remove
101.6	+

So now we are left with sample size $n = 14$.

We can base our test on either the + signs or the − signs. Let us choose + signs.

$X = 12$, we have 12 + signs.

+ sign indicates values > 98. So we need

$P(X \geq 12)$ from binomial with $n=14$ and $p = 0.5$

$P(X \geq 12) = 1 - \{P(0) + P(1) + \ldots + P(11)\}$ ~~$1 - P(X \leq 11)$~~, so we go to table for P(up to 11)

$= 1 - 0.9935 = 0.0065$ which is $< \alpha$

So we reject the null hypothesis.

Sign Test p. 448

Let us repeat but use the − signs as the criterion

The minus signs represent values < 98, and we have 2 of them, $x = 2$, so we need to find

$P(X \leq 2)$ from the binomial. with $n = 14$, $p = 0.5$

From table $P(X \leq 2) = 0.0065$

This value is < 0.01 so we reject our null hypothesis

— DONE ! —

What if we tested the following hypothesis

$$H_0 : \tilde{\mu} = 98$$

$$H_a : \tilde{\mu} < 98$$

$$\alpha = 0.01$$

Value	Sign
99	−
102.3	−
99.8	−
100.5	−
99.7	−
96.2	+
99.1	−
102.5	−
103.3	−
97.4	+
100.4	−
98.9	−
98.3	−
98.0	remove
101.6	−

So if less than 98 we give it + sign (we could have done opposite too)

$x = 2$ using + sign.

$$P(X \leq 2) = P(0) + P(1) + P(2)$$
$$= 0.0065$$

OR with − signs:

$$P(X \geq 12) = 1 - P(X \leq 11)$$
$$= 1 - 0.9935 = 0.0065$$

Wilcoxon Test p. 449

In this example we are testing a characteristic of two samples of sand if they have to same particle diameters.

H_0 : populations are the same

H_a : populations are not the same.

$\alpha = 0.01$. Note that this test is always set up as a 'one-tail' test.

Step 1. Combine the samples and rank each value, and note where the value came from.

Value	rank	mod rank	from
0.63	19		1
0.17	2		1
0.35	7		1
0.49	14		1
0.18	3		1
0.43	10		1
0.12	1		1
0.20	4		1
0.47	12		1
1.36	29		1
0.51	15		1
0.45	11		1
0.84	20		1
0.32	6		1

Value	rank	mod rank	from
0.40	9		1
1.13	28		2
0.54	17		2
0.96	24		2
0.26	5		2
0.39	8		2
0.88	21		2
0.92	23		2
0.53	16		2
1.01	25		2
0.48	13		2
0.89	22		2
1.07	26		2
1.11	27		2
0.58	18		2

We don't have any ties, so we dont need to calculate modified rankings. For example lets say we had ~~three~~ 4 values of 0.46 tied for the ~~fo~~ ranks 12, 13, 14, 15

Value	rand	mod rank	from
⋮			
0.46	12	13.5	1
⋮			
0.46	13	13.5	1
⋮			
0.46	14	13.5	1
⋮			
0.46	15	13.5	2

$$\text{modified rank} = \frac{12+13+14+15}{4} = 13.5$$

— back to question —

Step 2: Sum the ranks for each sample

$$W_1 = \sum_{i=1} \text{rank}_{i,1} = 162$$

$$W_2 = 273$$

Now calculate the U statistics

$$U_1 = W_1 - \frac{n_1(n_1+1)}{2} = 162 - \frac{15(15+1)}{2} = \cancel{57}\ 42$$

$$U_2 = 273 - \frac{14(14+1)}{2} = 168$$

Step 3: Calculate the mean and std dev of our U statistics,

$$\mu_{u_1} = \frac{n_1 n_2}{2} = \frac{15(14)}{2} = 105$$

$$\sigma_{\mu_1} = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}} = \sqrt{525} = 22.91$$

corresponding values for U_2 will be identical.

Step 4: Provided n_1 and n_2 are greater than 8, we can compute a test statistic as follows (otherwise, the test is not valid for your data).

$$Z = \frac{U_1 - \mu_{u_1}}{\sigma_{u_1}} = \frac{42 - 105}{22.91} = -2.749$$

or

$$Z = \frac{U_2 - \mu_{u_2}}{\sigma_{u_2}} = \frac{168 - 105}{22.91} = 2.749$$

Step 4: Reject H_0 if $Z < -Z_\alpha$ or $Z > +Z_\alpha$

so selecting $Z = -2.749$, $Z_{crit, 0.01} = -2.56$

We reject H_0. The two populations are identical!

Kruskal-Wallis Test

This the nonparametric counterpart of the Anova test

Example on p. 452.

In this example we are testing the anti-corrosion performance of three systems.

H_0: they all perform the same

H_a: at least one performs differently

$\alpha = 0.05$ (this is always a 'one-tail' test)

Step 1. We jointly rank them

A	77	54	67	74	71	66
rank	18	6	14	17	16	13

$R_i = \sum_i \text{rank}_A = 84$

B	60	41	59	65	62	64	52
rank	9	1	8	12	10	11	4 $\boxed{4.5}$

$R_B = 55.5$

C	49	52	69	47	56
rank	3	5 $\boxed{4.5}$	15	2	7

$R_c = 31.5$

Step 2: Compute H-statistic

$$H = \frac{12}{n(n+1)} \sum_{i=1}^{K} \frac{R_i^2}{n_i} - 3(n+1)$$

$$n = n_A + n_B + n_C = 6 + 7 + 5 = 18$$

we have $K = 3$ treatments.

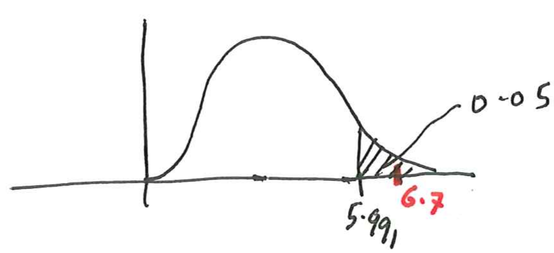
$$H = \frac{12}{18(19)} \cdot \left\{ \frac{84^2}{6} + \frac{55.5^2}{7} + \frac{31.5^2}{5} \right\} - 3(18+1)$$

$$= 6.7$$

Step 3: Now determine H_{crit} from chi-squared distribution ~~of~~ with $\chi^2_{\alpha=0.05, df=K-1=2}$

Our $\chi_{0.05, 2} = 5.991$

Step 4: our $H > 5.991$



We reject the null hypothesis. All systems do not work the same.

Spearman's Rank Correlation

Problem 14.10.a.

A	rank R_i	B	rand S_i	$R_i S_i$
0·15	3	0·75	5	15
0·30	6	0·60	3	18
0·20	4	0·80	6	24
0·00	1	0·50	1	1
0·10	2	0·55	2	4
0·25	5	0·70	4	20
0·40	7	0·95	7	49
	$\Sigma = 28$		$\Sigma = 28$	$\Sigma = 131$

Note that if we had any tie, we would apply the average rank to those tied values, as we have seen before.

rank correlation coefficient

$$r_s = \frac{\sum_{i=1}^{n} R_i S_i - n(n+1)^2/4}{n(n^2-1)/12}$$

note that here n = number of bivariate pairs.

$$r_s = \frac{131 - \frac{7(8^2)}{4}}{7(49-1)/12} = \frac{19}{28} = 0{\cdot}6785$$

fairly strong correlation.